

Thermal conductivity of metals

1. Overview of heat transfer phenomena

Heat transfer is the transition of thermal energy from a heated item to a cooler item. When an object or fluid is at a different temperature than its surroundings or another object, transfer of thermal energy, also known as heat transfer, or heat exchange, occurs in such a way that the body and the surroundings reach thermal equilibrium. Heat transfer always occurs from a hot body to a cold one, a result of the second law of thermodynamics. Where there is a temperature difference between objects in proximity, heat transfer between them can never be stopped; it can only be slowed down.

Classical transfer of thermal energy occurs only through conduction, convection, radiation or any combination of these. Heat transfer associated with carriage of the heat of phase change by a substance (such as steam which carries the heat of boiling) can be fundamentally treated as a variation of convection heat transfer. In each case, the driving force for heat transfer is a difference of temperature.

2. Heat conduction

Conduction is the transfer of thermal energy from a region of higher temperature to a region of lower temperature through direct molecular communication within a medium or between mediums in indirect physical contact without a flow of the material medium. The transfer of energy could be primarily by elastic impact as in fluids or by free electron diffusion as predominant in metals or phonon vibration as predominant in insulators. In other words, heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from atom to atom.

Conduction is greater in solids, where atoms are in constant close contact. In liquids (except liquid metals) and gases, the molecules are usually further apart, giving a lower chance of molecules colliding and passing on thermal energy. Heat conduction is directly analogous to diffusion of particles into a fluid, in the situation where there are no fluid currents. This type of heat diffusion differs from mass diffusion in behaviour, only in as much as it can occur in solids, whereas mass diffusion is mostly limited to fluids. Metals (eg. copper) are usually the best conductors of thermal energy. This is due to the way that metals are chemically bonded: metallic bonds (as opposed to covalent or ionic bonds) have free-moving electrons and form a crystalline structure, greatly aiding in the transfer of thermal energy. As density decreases so does conduction. Therefore, fluids (and especially gases) are less conductive. This is due to the large distance

between atoms in a gas: fewer collisions between atoms means less conduction. Conductivity of gases increases with temperature but only slightly with pressure near and above atmospheric. To quantify the ease with which a particular medium conducts, the **thermal conductivity**, also known as the **conductivity constant** or **conduction coefficient**, κ is used. This parameter is defined as the infinitesimal quantity of thermal energy, δQ , transmitted during an infinitesimal time interval dt through a thickness dx , in a direction normal to a surface of area (A), due to a temperature difference (dT):

$$\frac{\delta Q}{dt} \equiv q(x, t) = -\kappa A \frac{\partial T(x, t)}{\partial x} \quad (1)$$

where q is the so-called heat current. Eq. (1) is suitable to describe the heat transport along a specified direction (here denoted by x). In general three-dimensional cases, the spatial variation of the temperature must be taken into account and the temperature gradient should replace its partial derivative in the right hand side of Eq. (1). Accordingly, the heat current will become a vector quantity.

Thermal conductivity is a material property that is primarily dependent on the medium's phase, temperature, density, and molecular bonding. Eq. (1) is valid when the medium is homogeneous. When the heat transport occurs in inhomogeneous media, the thermal conductivity varies with x . A particularly interesting case is when $\kappa(x)$ has sharp discontinuities at interfaces between thermally different environments. In such cases, the heat current across the interface obeys the Newton's cooling law:

$$q(t) = -\eta A \Delta T, \quad (2)$$

where $\Delta T = T_1 - T_2$ is the temperature jump across the interface when the heat flows from region 1 towards region 2. The quantity η is the so-called **heat transfer coefficient** and is a characteristic of every thermal interface.

3. Example of a mixed problem with heat conduction and cooling phenomena

Consider a rod of a homogeneous material (e.g. a metal) placed in some atmospheric environment. The rod is heated at one of its ends and the thermal energy propagates along it towards the opposite end. Simultaneously, a cooling process takes place at every segment of the rod. If the heat source is constant in time, a stationary temperature distribution will be achieved along the rod. By focusing on some infinitesimal segment of the rod, between x and $x + dx$, one may easily express the *balance of thermal energy in the steady state* in the following way: the heat $q(x)dt$ entering the segment through its end that is closest to the heat source will split in $q(x+dx)dt$ that is delivered through the other end and the heat $dq(x)dt$ that is lost to the neighboring air by the cooling process (Fig. 1).

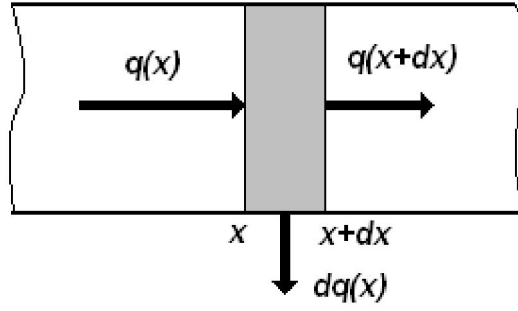


Figure 1: Sketch of the steady-state thermal energy balance in an infinitesimal segment of the test rod.

Writing $T'(x)$ for the position derivative of the temperature (which is now constant in time), one may apply Eq. (1) to both ends of the considered segment:

$$\left. \begin{array}{l} q(x+dx) = -\kappa A T'(x+dx) \\ q(x) = -\kappa A T'(x) \end{array} \right\} \Rightarrow q(x+dx) - q(x) = -\kappa A [T'(x+dx) - T'(x)].$$

By further applying well-known rules of Calculus, one may find:

$$dq(x) = -\kappa A T''(x) dx. \quad (3)$$

The heat current $dq(x)$ is lost by cooling to the environment. Therefore Eq. Can now apply with A replaced by the lateral area $\pi d dx$ of the considered segment and with ΔT replaced by $T(x) - T_a$, where T_a is the environmental air temperature and d is the diameter of the assumed cylindrical rod:

$$dq(x) = -\pi d \eta [T(x) - T_a] dx. \quad (4)$$

Combining Eqs. (3) and (4), the following differential equation for the stationary temperature distribution along the rod is readily obtained:

$$T''(x) - \alpha^2 [T(x) - T_a] = 0; \quad \alpha = \sqrt{\frac{4\eta}{\kappa d}}. \quad (5)$$

This second-order differential equation has a straightforward general solution:

$$T(x) = T_a + A e^{\alpha x} + B e^{-\alpha x}, \quad (6)$$

where A and B are undetermined constants. To specify these constants, the values of the temperature at two different positions on the rod should be known. For example, take the temperatures at the ends of the rod, T_h and T_l , as given: $T(x=0) = T_l$ and $T(x=L) = T_h$. Applying these conditions to the solution (6), one may readily find:

$$A = \frac{1}{2 \sinh(\alpha L)} [T_h - T_l e^{-\alpha L} - T_a (1 - e^{-\alpha L})], \quad (7)$$

$$B = \frac{1}{2 \sinh(\alpha L)} \left[-T_h + T_l e^{\alpha L} - T_a (e^{\alpha L} - 1) \right], \quad (8)$$

where $\sinh(\varphi)$ is the hyperbolic sinus of the quantity φ .

4. Method of measuring the thermal conductivity of a metallic rod

The conditions described in the previous section can easily be obtained for a sample metallic rod. A constant flux of thermal energy can be injected through one of its end and the stationary temperature distribution can be measured along the rod in some specified points. Thus, an experimental steady-state temperature distribution, $T_{exp}(x)$ is obtained. By modifying the parameter α in Eqs. (6)-(8), one may fit the experimental curve with the theoretical one. From the best-fit value of α one may then find the thermal conductivity of the sample (Eq. (5)). The only problem is to have the corresponding heat transfer coefficient, η , and the geometrical parameters of the rod.

The heat transfer coefficient can be determined separately, by a direct cooling experiment with the whole rod initially heated at some temperature T_0 . The rod is then placed for cooling in the same environment as before. Its average temperature is measured over time and the theoretical cooling curve, $T(t)$, is fitted with η as a parameter on the experimental one. For the present version of the experiment the value of η for the (aluminium) rod has been determined: $\eta = 15,129 \text{ W}/(\text{m}^2\text{K})$.

5. Experimental setup

The sample rod is placed vertically with its upper end in tight contact to an aluminium receptacle where water is kept at the boiling point by means of an immersion heater (Figure 2).

Warning! Keep clear of the receptacle's area to avoid burns from erratic drops of hot water jumping out the vessel.

In the current version of this experiment, the rest of the rod is in full contact with the environmental air whose temperature can be measured with a thermocouple fixed in rod's neighborhood. Another thermocouple is used to measure the local temperature by placing it in special cavities of the rod. To allow better thermal contact that thermocouple's end should be greased with special thermo-conductive paste. Temperatures are read in Celsius on a two-screen universal measuring amplifier.



Figure 2: Experimental set-up for thermal conductivity.

The cavities positions with respect to the lower end of the rod are the following: 0 cm (just at the lower end); 7,0 cm; 10,4 cm; 14,0 cm; 17,5 cm; 21,0 cm; 24,4 cm; 28,0 cm; 31,5 cm; 35,0 cm; 38,5 cm. Astfel, $L = 38,5$ cm.

6. Procedure

The temperature at the lowest end of the rod is measured for some time, about 10 minutes after the water became to boil in the receptacle. If the measured values seem to keep constant in time, the steady-state of the cooling phenomenon is reached. From this moment on the temperature distribution can be read by placing the greased end of the thermocouple in each alveolus of the rod, starting from the very bottom one. The temperatures are recorded and then written in a text file by means of the Notepad utility. The temperatures should be written one in a line by pressing “Enter” after each value. The text file should be then saved as “texp.txt” in the folder entitled “Conductivitate metale”, where the Mathcad 14 program “Conductivitatea termica a metalelor.xmcdz” exists. After completing the text file open the program “Conductivitatea termica a metalelor.xmcdz”. The program automatically computes the theoretical temperature distribution (red curve) of Eq. (6) for a given value of the parameter α . Also, the program reads the “texp.txt” file and plots the experimental temperatures (in Kelvin – blue dots) along with the theoretical distribution. In the first yellow placeholder defining the value of the variable “taer” write the average value of the air temperature near the sample rod during the experiment. Then modify the parameter α in the second yellow placeholder to get the theoretical temperature distribution as close as possible to the experimental one. The procedure is termed as *manual fitting*. To run the program each time a modification of the value of α has been made one can simply move the cursor in line with the graph and left click the mouse (alternatively, the key F9 can be pressed). When the best fit has been attained, read the corresponding value of the thermal conductivity in the brick placeholder in line with the graph. The thermal conductivity is computed from the value of α from the following relation (see Eq. 5):

$$\kappa = \frac{4\eta}{\alpha^2 d}. \quad (9)$$

Notes

- 1) The value of α to be entered in the second yellow placeholder of the fitting program is actually a dimensionless parameter. The one used in Eq. 9 can be obtained from it through a multiplication by the sample’s length, L .
- 2) Due to the probable bad thermal contact during the temperature measurements, the method is prone to a systematic erroneous decrease of the thermal conductivity.